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ABSTRACT

Approximately 1,700 University of Illinois Committee on School Mathematics (UICSM) algebra students were divided into 6 groups depending on grade (8th or 9th), version of UICSM first course text used (1958 or 1959), and duration of study. These students and nearly 700 non-UICSM algebra students were given Cooperative Algebra Test (Coop Algebra) Forms T, X, and Y. Analysis of covariance was used to compare group means on this test, with adjustments for inequalities between groups on the Differential Aptitude Tests of Numerical Reasoning and of Verbal Ability. The UICSM students showed greater achievement than did the non-UICSM students. This was also true of four of the six UICSM groups taken separately. The adjusted Coop Algebra means of the other two UICSM groups were not significantly different from that of the control sample. Possible reasons for these results are discussed. (Author/BB)

UICSM

RESEARCH REPORT

UNIVERSITY OF ILLINOIS COMMITTEE ON SCHOOL MATHEMATICS

University of Illinois Committee on School Mathematics

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UICSM

RESEARCH REPORT

No. 1 September, 1963

Comparison of UICSM vs. "Traditional" Algebra Classes on Coop Algebra Test Scores

Maurice M. Tatsuoka and J. A. Easley, Jr.

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Foreword

The problems of evaluating new curriculum materials such as those produced by UICSM are manifold and not easy to solve. A primary objective which UICSM set for itself when it was organized in 1951 was to improve the learning of the skills and facts comprising the college preparatory sequence found in most high schools at that time. In trying to achieve this objective, UICSM made major changes in pedagogy and major changes in content especially with respect to logical organization. Teaching-by-discovery replaced teaching-by-telling in order to build interest and to tap the creative talents of students. We maintained very high standards of precision of language in our instructional materials. In order to achieve the understanding which intellectual integrity demands as a prerequisite to the acquisition of skills, we introduced the notion of a deductive organization of elementary algebra. Thus students derived the theorems which justified the usual manipulation rules and they became aware of the logical connections among the manipulation rules [e.g., the rules for combining like monomials and the rule for combining like radicals are logical consequences of the same principle]. Topics from the elementary theory of sets were introduced to give students insights into what made the mechanical procedures work when they solved equations and inequations or when they solved systems of equations by graphing. Thus UICSM students had the opportunity not only to acquire the skills and knowledge expected from the "traditional" curriculum but also to learn mathematical ideas and techniques which were not provided for in school mathematics programs prior to UICSM. In addition, as we were told by cooperating teachers in experimental classes and as we observed on our visits to these classrooms, UICSM students were routinely developing insights into mathematics and attitudes toward new problems which were acquired by only the best students in the best of traditional classes and which mathematics educators have professed to be among the most significant outcomes of instruction.

Any comprehensive evaluation program should consider both the primary skills-and-knowledge objective which is common to both our program and the traditional one, and the rather astounding secondary objectives which we appeared to accomplish almost as by-products. In the early years of our work we were able to look only at the primary objective in making comparative studies of UICSM students and students trained in the traditional curriculum. Naturally, it was not possible to make comparative studies with respect to new content, and we had neither the skill nor the time to develop objective tests for making comparative studies with respect to the development of insight and creativity in attacking new problems. What we could do and what we did was to collect data on students' achievement as measured by standardized algebra and geometry tests which were designed for the pre-1950 traditional curriculum. We do not pretend that these tests have provided us with a complete description of our students' achievement, and we recognize that the language used in many of the items on these tests was somewhat unfamiliar to our students. Nevertheless, we were willing to use the test results as a rough guide in revising our instructional materials. The study reported below is the first we have published based on an analysis of the data collected by means of such tests.

The results of the analysis are not as satisfactory as we might have hoped. Achievement by students in our pilot and participating schools was not uniformly high. These results are in accord with the subjective reports we have received from teachers and with very rough analyses which we carried out some years ago as the data were collected. These preliminary findings persuaded us to revise our instructional materials for our advanced units in the direction of including many review sections of miscellaneous manipulative exercises. The present study will be useful to the extent that the objectives measured by the tests are valid in our continuing curriculum development work.

The report which follows is the first to be produced by the newly-organized research section of UICSM. It is directed to a varied audience--research workers in mathematics education, teachers, administrators, and others who have been interested in UICSM. Such variety posed a problem for the authors. Professors Tatsuoka and Easley have felt obliged to include sufficient details of the statistical analysis to answer the research workers' questions but, at the same time, did not wish to overwhelm the nonstatistically oriented reader. They have, consequently, employed the device of setting off the technical sections with wider margins and closer spacing, so that the general reader may skip these if he wishes. They have also provided an abstract for those who wish a brief resume.

Our research section, under the direction of Professor Easley, is presently engaged in further analysis of available data, in collecting new data for studies underway, and in designing further studies related to the teaching and learning of mathematics. These research activities will play an important role in the development of new curriculum materials which is the primary job of UICSM.

Max Beberman
Director, UICSM

Urbana, Illinois
September 1, 1963

Comparison of UICSM vs. "Traditional" Algebra Classes on Coop Algebra Test Scores[1]

Maurice M. Tatsuoka and J. A. Easley, Jr.[2]

Abstract

This report describes the results of an evaluation study involving approximately 1700 UICSM and nearly 700 non-UICSM algebra students. The former are designated the "experimental" sample and the latter the "control" sample. The experimental sample was broken down into six groups depending on grade (8th or 9th), version of UICSM First Course text used (1958 or 1959), and duration of study prior to testing.

Achievement in algebra was measured by the Cooperative Algebra Test (Elementary), Forms T, X, and Y. The analysis of covariance was used in comparing group means on this test, with adjustments for inequalities between groups on the Differential Aptitude Tests of Numerical Reasoning and of Verbal Ability.

With these adjustments, the experimental sample as a whole showed significantly greater achievement than the control sample. This was also true of four of the six experimental groups taken separately.

The adjusted Coop Algebra means of the other two groups were not significantly different from that of the control sample. Possible reasons for these results are discussed in the report.

Description of Samples Tested in the Study

The experimental sample comprised 1,705 students in 75 eighth- and ninth-grade classes taking the UICSM First Course (Units I-IV) during the 1958-59 and 1959-60 school years. They fall in the upper two-thirds of the college preparatory students in 38 schools scattered throughout the country. The samples were not arrived at by stratified sampling but by

[1]Most of the data used in this paper were originally obtained and subjected to a different type of analysis by O. Robert Brown, Jr. in an unpublished report of the same title.

[2]The authors are especially indebted to Judy Boyle for compiling the data and assisting in the calculations and to Eleanor McCoy for compiling a list of changes made in the latest version of the UICSM first course text. We are also indebted to Robert Kansky for a number of helpful comments on the MS.

historical accident, from schools which volunteered to try the UICSM materials. (Due to this fact, it was necessary to introduce statistical controls, as described in the section on "Statistical Analysis.") This sample was divided into six groups (designated by E_1, E_2, E_3, E_4, E_5, and E_6), depending on the grade, the school year in which the UICSM First Course was taken, and the time of testing. Groups E_2 and E_3 contained 25% of the UICSM students starting First Course in September, 1958, and the other four represented 43% of the students starting in September, 1959.

The particular year (1958-59 or 1959-60) in which the course was taken is important because the UICSM material published and first used in the fall of 1959 was the result of a revision which, it was hoped, would improve student performance on standard high school algebra tests. While earlier versions of the UICSM texts had concentrated primarily on making ideas clear, the 1959 revisions reflected the realization that clarity of concept development was not enough to ensure that concepts are retained by students in a useful form. Although further improvement of clarity of concepts was not avoided in making these revisions, the principal purpose for the revisions was to provide increased practice in using mathematical ideas. The diversity of ways in which opportunities for practice were increased in the 1959 versions of Units I-III exemplified by the new topics and treatments briefly described below.

In Unit I, the explanation of and the exercises on the grouping conventions were considerably expanded, and the term "unabbreviating" was introduced to refer to the procedure of putting omitted symbols back into an expression. The treatment of some of the principles and concepts relating to real numbers was made more explicit, and more exercises were included which involved the use and recognition of basic principles for real numbers. In Unit II, the treatment of simplification problems involving division was considerably expanded and the total number of supplementary exercises was approximately doubled. The new exercises were mainly concerned with proofs and simplification problems involving division--in contrast to the old supplementary exercises (still retained) which concerned only substitution and algebraic simplification. In Unit III, the application of uniqueness and cancellation principles to the solution of linear equations and "inequations," quadratic equations and square roots received a more detailed treatment.

Qualitatively, teachers and writers were satisfied that these changes would promote an increased mastery of algebraic skills by the pupils. It is most interesting, therefore, to inquire whether students using the new edition showed such an increased mastery, as measured by a test emphasizing manipulative skill.

The time of testing for achievement in the experimental sample was rather disuniform from one class to another. Some classes (those in groups E_2 and E_4) were tested at the end of the school year in which they had started their UICSM courses, and most of them had just begun their study of Unit 4 at the time of testing. These students were at a disadvantage in that the criterion tests (described later) included items on exponents and on geometrical applications of algebra--topics which were not covered in Units 1-3. Other classes were tested upon completion of Unit 4, which generally took place some time during the fall of the school year following the one in which they had begun the course. The distribution of times of testing of the various classes is indicated graphically in Figure 1, which also shows the size of the groups into which the experimental and control samples were divided.

Note that the year and month in which each group began its algebra course is indicated by the position of the left border of its block along the

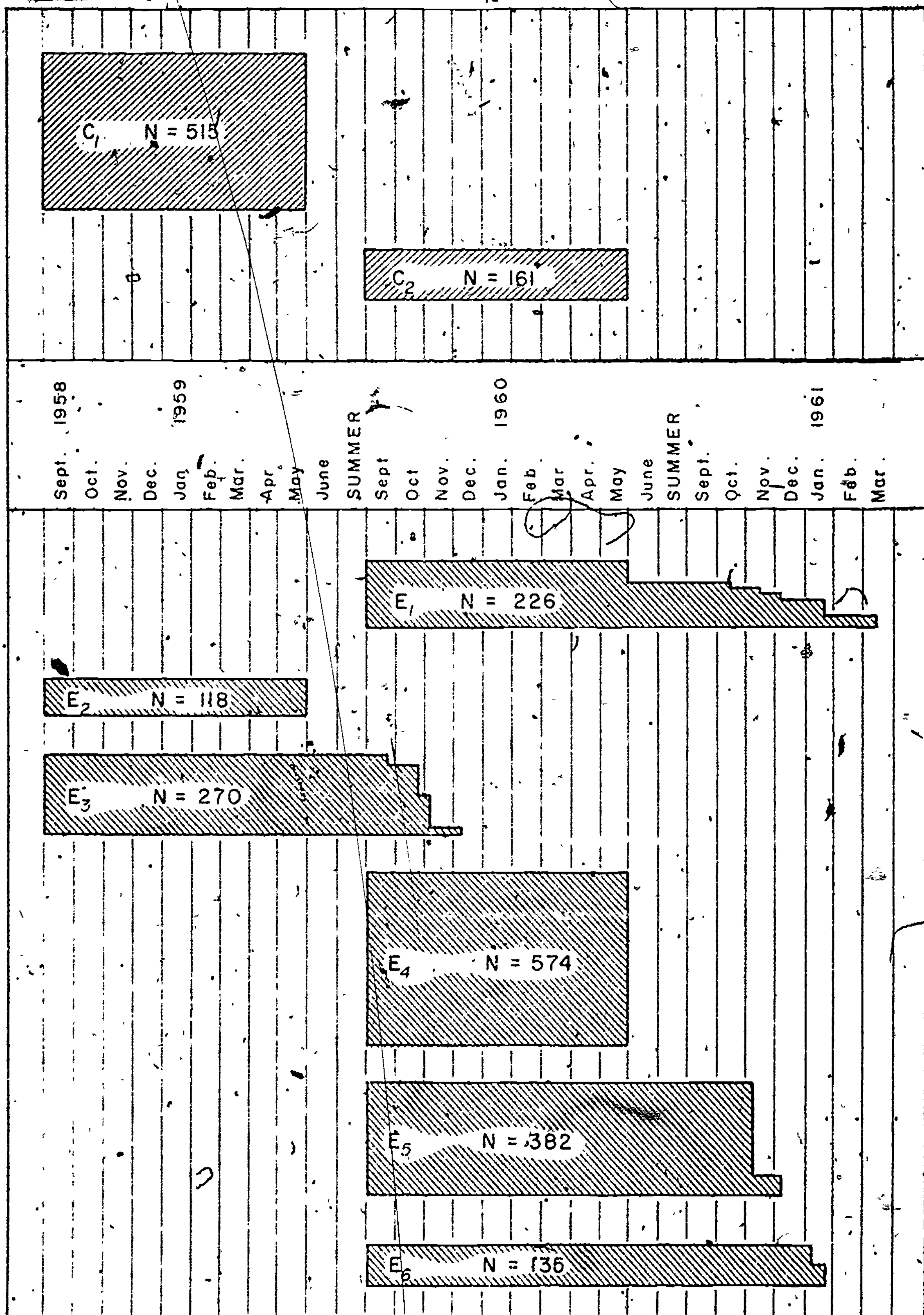

Figure 1

Block diagram showing the groups into which the control and experimental samples were divided. The left edge of each block indicates the time at which that group began its course. The distribution of times of testing for these groups is indicated by the right hand boundaries of the blocks.

time scale separating the blocks representing the control and experimental groups. The right-hand border indicates the time of testing with the Coop Algebra Test. For those groups which contained classes that varied in time of testing, the right-hand border is made step-wise to indicate the variations. The heights of these steps are drawn roughly proportional to the number of students tested in each time period; the total height of each block is proportional to the group size.

The breakdown of the experimental sample into six groups, in accordance with the characteristics described above, is summarized in the upper portion of Table 1, below:

Table 1. Subdivisions of experimental and control samples into several groups based on some relevant features.

	Group	Grade	Year Course Was Begun	Time of Testing	N
Experimental Sample	E_1	8	1959	May '60-Mar '61	226
	E_2	9	1958	May '59	118
	E_3	9	1958	Sept - Dec '59	270
	E_4	9	1959	May '60	574
	E_5	9	1959	Nov - Dec '60	382
	E_6	9	1959	Jan '61	135
Control Sample	C_1	9	1958	May '59	515
	C_2	9	1959	May '60	161

The control sample comprised 676 students in 26 ninth-grade classes taking "conventional" algebra courses during 1958-59 and 1959-60 in schools which also had UICSM classes. The control classes were from 11 of the 38 schools represented in the experimental sample and 10 others. (All schools using UICSM text were encouraged to set up both experimental and control classes, but many were unable to do so or failed to administer all the tests.) This sample was divided into only two groups as shown in the lower portion of Table 1, (C_1: the 1958-59 classes; C_2: the 1959-60 classes). Further subdivision was unnecessary because, unlike those in the experimental sample, the control classes were uniform with respect to grade and time of testing. That is, all control classes were 9th-grade classes, and all were tested at the end of the school year in which they had their first algebra course.

Test Data Used

The criterion variable in terms of which intergroup comparisons were made was the Cooperative Elementary Algebra Test (Forms T, X, and Y), published by the Educational Testing Service. This test emphasizes computational skills, and hence provides an "acid test" for UICSM-trained students--especially for those who were taught with material prior to the 1959 revision.

The means and standard deviations of the Coop Algebra Test scores for the experimental and control samples, as well as for the several subgroups, are shown in Table 2, below. It is seen that the experimental (UICSM) sample as a whole (and each subgroup thereof) had a higher mean score than either of the control ("conventional") subgroups. The superior performance of UICSM-trained students must, however, be partly attributed to the selection factor: they represented the upper two-thirds of

Table 2. Means and Standard Deviations of the Coop Algebra Tests,* DAT-VR, and DAT-NA for the experimental and control samples and the several subgroups thereof. (Also shown are the coefficients of multiple correlation of Coop Algebra on the two DAT subscores.)

	Total Experimental		E_1		$E_2E_3E_4E_5E_6$		E_6	
TEST	Mean	S. D.	Mean	S. D.	Mean	S. D.	[illegible]	S. D.
Coop Algebra	63.37	10.19	66.89	10.03	62.83	10.10	66.58	7.94
DAT-VR	26.98	8.11	27.39	7.61	26.92	8.18	28.42	7.51
DAT-NA	24.62	7.19	23.72	7.90	24.76	6.98	26.14	5.62
Multiple r	.645		.670		.653		.602	

	E_3		E_4		E_5		E_6	
TEST	Mean	S. D.	Mean	S. D.	Mean	S. D.	Mean	S. D.
Coop Algebra	63.72	8.48	59.57	11.22	65.83	8.61	63.18	9.50
DAT-VR	28.57	7.86	24.22	8.31	28.62	7.68	28.95	7.02
DAT-NA	25.69	6.68	22.95	7.66	26.04	6.39	25.76	5.51
Multiple r	.660		.671		.527		.649	

	Total Control		C_1		C_2	
TEST	Mean	S. D.	Mean	S. D.	Mean	S. D.
Coop Algebra	58.26	11.09	58.77	11.33	56.65	10.11
DAT-VR	23.63	8.02	23.46	8.23	24.19	7.27
DAT-NA	21.26	7.20	21.66	7.38	23.09	6.48
Multiple r	.673		.718		.569	

* The norms for this test, based on 15,000 students in 130 schools, show a mean of 52.9 with a standard deviation of 9.6.

the college preparatory students in each school, as mentioned earlier. On the other hand, the control sample was formed from algebra classes for which the admission requirements were generally not so stringent.

In order to make adjustments for the inequality in "scholastic ability" between the experimental and control samples, the Differential Aptitude Test Battery (DAT) results for all individuals in our sample were employed. This battery had been administered prior to the students' taking their algebra courses, UICSM or "conventional", as the case may be. Specifically, the scores on the two subtests--Verbal Reasoning (VR) and Numerical Ability (NA)-- of the DAT were used as statistical control variables as described in the next section. The means and standard deviations of DAT-VR and DAT-NA for the various groups are shown in Table 2. As to be expected, the experimental groups have higher means than the control groups.

It is recognized that adjustments for DAT-VR and DAT-NA scores are probably insufficient to compensate for all the relevant but uncontrolled inequalities existing between experimental and control samples. For instance, the "teacher variable" is quite likely to be a significant factor contributing to the difference in algebraic achievement between the two samples. But the only measure available for assessing "teacher competence" was the very crude one of the number of years of experience, and it was decided that little would be gained by making adjustments for this variable. The adjustments which we made, therefore, probably represent the most that could be done within the limits of available data.

Some indication of the extent to which these adjustments were effective can be gained by examining the magnitudes of the coefficients of multiple correlation of Coop Algebra scores on the two DAT subscores. These ranged from about

.53 to .72, depending on the particular group, as shown in Table 2. Thus, it can be argued that the two DAT scores accounted for from 30 to 50 percent of the variance in the Coop scores.

Statistical Analysis

The statistical technique used for making a majority of the intergroup comparisons described below was the standard analysis of covariance. This technique amounts to making an estimate of what the Coop Algebra mean of each group under comparison would have been if the DAT-VR and DAT-NA means had each been equal for the several groups, and then making a comparison between these estimated (or "adjusted") Coop Algebra means.

In actual practice, it is the sums-of-squares of the Coop Algebra tests that are adjusted. The test of significance then exactly parallels that for the usual analysis of variance. That is, the ratio of the adjusted between-groups mean-square to the adjusted within-groups mean-square is used as an F-ratio. If the value of this ratio exceeds a critical value, the adjusted Coop Algebra means are judged to be significantly different from each other.

In this manner, comparisons were made between the experimental sample as a whole and the control sample as a whole; between groups E_2 through E_6 (9th grade classes in experimental sample) and the control sample; and so forth. Each comparison was designed to test the effect of some particular variable or combination of variables on which the contrasted groups differed systematically.

Experimental Sample vs. Control Sample: The analysis-of-variance table for the adjusted sum-of-squares (S.S.) in this comparison is shown in Table 3, as are the adjusted means.

Table 3. Adjusted analysis of variance table for comparing experimental group and control group sample Coop Algebra means

Source	Adjusted S. S.	n. d. f.	Adjusted Mean-Square	F-ratio
Between-groups	1944.83	1	1944.83	31.19*
Within-groups	148214.00	2377	62.35	
Total		2378	---	

Adjusted Coop Algebra means	Group E_{total} = 62.50 Group C_{total} = 60.45	*P. < .001

The resulting F-ratio (31.19) indicates that the adjusted Coop Algebra means for the two samples are significantly different at the .001-level, the direction of the difference being in favor of the experimental (UICSM) sample. It should be noted that the adjustments for inequalities on the two DAT means have lowered the experimental sample's Coop Algebra mean from 63.37 (cf. Table 2) to 62.50, and raised the corresponding control-sample mean from 58.26 (cf. Table 2) to 60.45. This is the way in which the analysis of covariance method operates in order, at least partly, to compensate for group inequalities on related variables.

We may thus conclude that the UICSM sample as a whole showed a significantly higher algebra achievement (as measured by the Cooperative Elementary Algebra Test) than did the "conventional" algebra sample as a whole--even when due compensations are made for the fact that the former was superior to the latter in general academic ability as measured by the DAT Verbal Reasoning and Numerical Ability scores.

Group E_1 vs. Groups $E_{(2.\ldots.6)}$. In view of the fact (cf. Table 2) that the 8th-grade experimental classes (group E_1) scored considerably higher on the Coop than did the 9th-grade experimental classes (groups $E_{(2.\ldots.6)}$), it is pertinent to inquire whether the means for these two sub-samples are significantly different. For if so, there arises the possibility that the superiority, seen above, of the experimental sample is attributable to just the 8th-grade segment, which constitutes about 13% of the experimental sample.

The adjusted analysis-of-variance table for the experimental 8th- vs. 9th-grade comparison is shown in Table 4. The resulting F-ratio indicates a significant difference in favor of the experimental 8th-grade classes.

> Note that, in this case, the adjustments have widened the gap between the group means under comparison. Reference to Table 2 shows that the 8th graders had a lower DAT-NA mean than the 9th graders, although the DAT-VR mean of the former was higher than that of the latter. Hence, the adjustments for the two predictors worked in opposite directions in this case, and the fact that the 8th-grade mean was raised from 66.89 to 67.33 while that of the 9th graders was lowered from 62.83 to 62.76 indicates the overriding effect of the compensation for unequal Numerical Ability scores.

At any rate, since the 8th-grade classes were found to be significantly superior to the 9th-grade experimental classes on the Coop Algebra Test, it is desirable to inquire whether or not the superiority of the experimental to the control sample will continue to hold when the 8th-grade group is removed from the experimental sample.

Experimental Groups $E_{(2.\ldots.6)}$ vs. Control Sample. That the answer to the question posed above is in the affirmative can be seen from the results presented in Table 5. The experimental sample shows a significantly higher overall performance on the Coop Algebra Test than the control sample, even after the especially high-scoring experimental group (the 8th-grade classes) is removed from consideration.

Table 4. Adjusted analysis of variance table for comparing Group E_1 and Groups $E_{(2.....6)}$ means on Coop Algebra

Source	Adjusted S.S.	n.d.f.	Adjusted Mean-Square	F-ratio
Between-groups	2027.47	1	2027.47	34.79*
Within-groups	99,082.24	1701	58.25	
Total		1702		

Adjusted Coop Algebra means	Group E_1	= 67.33	*P < .001
	Group $E_{(2.....6)}$	= 62.76	

Table 5. Adjusted analysis of variance table for comparing experimental groups $E_{(2.....6)}$ and control-sample means on Coop Algebra

Source	Adjusted S.S.	n.d.f.	Adjusted Mean-Square	F-ratio
Between-groups	911.21	1	911.21	14.90*
Within-groups	131,504.11	2151	61.14	
Total		2152	---	

Adjusted Coop Algebra means	Groups $E_{(2.....6)}$	= 61.85	*P < .001
	Group C_{total}	= 60.04	

<u>Individual Experimental Groups vs. Control Sample.</u> We now compare each of the experimental 9th-grade groups (E_2, E_3, E_4 and E_5), in turn, with the control sample. Group E_6 was not considered because it was both very small in size (N = 135) and extreme in its delayed testing time, as seen in Table 1. The results of the four comparisons are summarized in Table 6, showing that two of the experimental groups (E_2 and E_5) were found to be significantly superior to the control sample while two (E_3 and E_4) were not significantly different from it.

> It should be pointed out that, for the comparison involving group E_5, an unrestricted linear hypothesis model (using separate group regression weights) was used instead of the standard covariance analysis.[2] This is because the equality-of-slopes condition, prerequisite to regular covariance analysis, was not satisfied in this case.

The two experimental groups that failed to achieve adjusted Coop Algebra means which were significantly higher than that of the control sample apparently did so for two different reasons. Group E_3 classes, having taken their UICSM course prior to the 1959 revision of materials, were at a disadvantage as noted earlier, in not having sufficient drill in the manipulative skills. Group E_4, on the other hand, was tested before completion of Unit 4, and hence had not yet covered some of the topics that were included in the Coop test.

Group E_5 was in the most advantageous position of the four experimental groups: these classes were taught with the revised material which contained more exercises in computation, and they were tested after

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[2] We are indebted to Dr. Frank Watson of the Office of Instructional Television, University of Illinois, for making available to us his program for carrying out this test, and to Mr. James Hennes for its actual execution on the IBM 7090 computer.

Table 6. Comparison of experimental groups E_2, E_3, E_4, E_5, each with control sample, on Coop Algebra means

(a) Group E_2 vs. Group C_{total}

Source	Adjusted S. S.	n. d. f.	Adjusted Mean-Square	F-ratio
Between-groups	1103.91	1	1103.91	17.57*
Within-groups	49639.25	790	62.83	
Total		791	---	

Adjusted Coop Algebra means: Group E_2 = 62.41; Group C_{total} = 58.99

*P < .001

(b) Group E_3 vs. Group C_{total}

Source	Adjusted S. S.	n. d. f.	Adjusted Mean-Square	F-ratio
Between-groups	150.53	1	150.53	2.53*
Within-groups	56011.35	942	59.46	
Total		943	---	

Adjusted Coop Algebra means: Group E_3 = 60.48; Group C_{total} = 59.56

*not signif.

Table 6. (continued)

(c) Group E_4 vs. Group C_{total}

Source	Adjusted S.S.	n.d.f.	Adjusted Mean-Square	F-ratio
Between-groups	45.21	1	45.21	*
Within-groups	84399.96	1246	67.74	
Total		1247	----	

Adjusted Coop Algebra means	Group E_4	= 59.07	*not signif.
	Group C_{total}	= 58.68	

(d) Group E_5 vs Group C_{total}

Difference between adjusted means

(Group E_5 minus Group C_{total}) 3.00

standard error of difference .538

$t = 5.57$ $(P < .001)$

completion of Unit 4. It would have been quite discouraging for UICSM if this group had not excelled over the control sample; but it did.

The success of Group E_2 (which combined the disadvantage of being taught with pre-1959 material and of being tested promptly at the end of the school year), on the other hand, is rather remarkable. At least part of this success must be attributed to the group's exceptional superiority in general scholastic ability--as indicated by its being considerably above the overall experimental-sample average on DAT-VR, and the highest of all the groups on DAT-NA (cf. Table 2). But there is also other evidence indicating that more than one of the classes in this group had highly competent teachers who successfully covered almost the entire first course in one school year. This observation lends support to the surmise that, given these favorable conditions, the UICSM material, even without augmentation of computational drill exercises, was adequate in preparing superior students to cope with such conventionally oriented achievement tests as the Coop Algebra.

Experimental Groups vs. Control Group C_1. Prior to making the above comparisons between each experimental group, in turn, with the entire control sample, we undertook to compare some of the experimental groups with control group C_1. This group has a somewhat higher mean Coop Algebra score (58.77) than the other control group (56.65), and we, therefore, thought that comparisons with group C_1 would offer a more stringent test than comparisons with the entire control sample. However, an obstacle was encountered to pursuing this plan, which we shall describe below, and it was therefore decided to use the entire control sample as the basis of comparisons.

The difficulty with using control group C_1 alone for comparisons stems from the fact that the regression plane of Coop Algebra on the two DAT subtests for this group were markedly non-parallel to those of most of the experimental groups. Thus, the equality-of-slopes condition, mentioned earlier, would not be satisfied, and the standard analysis of covariance method for comparing adjusted means would not be applicable. There are two alternative procedures that can be used in such cases, both of which are based on an unrestricted linear hypothesis model. One of these procedures was used for the group E_5 versus control sample comparison, cited earlier. The other is the Johnson-Neyman technique, which enables us to specify three regions in the covariate space (in this study the two-dimensional space with DAT-VR and DAT-NA as the coordinate axes), such that the conclusion to be drawn depends on the region(s) in which the centroids of the two groups under comparison fall. If both centroids fall in, say, ~~Region~~ A, we conclude that group A has a significantly higher adjusted criterion mean than group B; if both centroids fall in Region B, the reverse is true; and if one or both of the centroids fall in the third region, the difference is not significant.

Unfortunately, it appears that the two procedures may sometimes lead to conflicting results, a case at point being the following conclusions, (1) and (2) below, for the comparison between experimental group E_3 and control group C_1:

(1) Separate-group regressions:
Difference between adjusted Coop means = 6.92
(in favor of group E_3)
Standard error of difference = 2.33
$t = 2.97$ ($P < .01$)

(2) Johnson-Neyman technique: the C_1 group centroid falls in the region favoring group E_3, and the E_3 centroid falls in the region of non-significance.

Thus, it seems that, when the equality-of-slopes condition is not satisfied, we may not be able to draw unequivocal conclusions as we can when the standard covariance analysis is applicable. When the entire control sample was considered, the regression plane was not significantly non-parallel to that of most of the experimental groups. The single exception was the case of group E_5, already discussed, and it should be mentioned here that for this comparison the Johnson-Neyman technique led to the same conclusion as that from the separate-group-regressions approach.

<u>Group E_2 vs. Group E_4; Group E_3 vs. Group E_5.</u> The last two comparisons to be reported are those between pairs of groups in the experimental sample itself. Specifically, the purpose was to compare the 1958 (pre-revision) and 1959 (post-revision) classes, holding relatively constant

the amount of the course covered as of the time of testing. Reference to Table 1 shows that groups E_2 and E_4 (end-of-year testing) and groups E_3 and E_5 (end-of-course testing) are the appropriate pairs to compare. The results are presented in Table 8.

Table 8. Comparisons within the Experimental Sample

(Ninth Grade Groups)

(a) Group E_2 vs. Group E_4

Source	Adjusted S.S.	n.d.f.	Adjusted Mean-Square	F-ratio
Between-groups	1012.59	1	1012.59	15.63*
Within-groups	44,581.80	688	64.80	
Total		689	---	

Adjusted Coop Algebra means	Group E_2 = 63.49 Group E_4 = 60.20	*$P < .001$

(b) Group E_3 vs. Group E_5

Source.	Adjusted S.S.	n.d.f.	Adjusted Mean-Square	F-ratio
Between-groups	577.10	1	577.10	11.86*
Within-groups	31,524.05	648	48.65	
Total		649	---	

Adjusted Coop Algebra means	Group E_3 = 63.21 Group E_5 = 65.75	*$P < .01$

It was already mentioned above that group E_2 was an exceptionally superior group. It is not surprising, therefore, that it excelled over its 1959 counter-part, group E_4, despite the latter's having been exposed to a larger number of drill exercises.

On the other hand, of the two groups which were tested after completion of the course, group E_5 (1959) was significantly superior to group E_3 (1958) in achievement. Thus, we find at least tentative evidence that the 1959 revision was favorable to better preparing UICSM students to achieve those skills that are stressed in conventional algebra courses.

Summary of Results

Since many different comparisons were reported to the foregoing, it may be helpful to present an overview that will enable the reader to see the results at a glance. We do this in two ways. First, Figure 2 shows, in ascending order, the several group means on Coop Algebra, adjusted by a regression equation based on the entire sample (experimental and control combined).

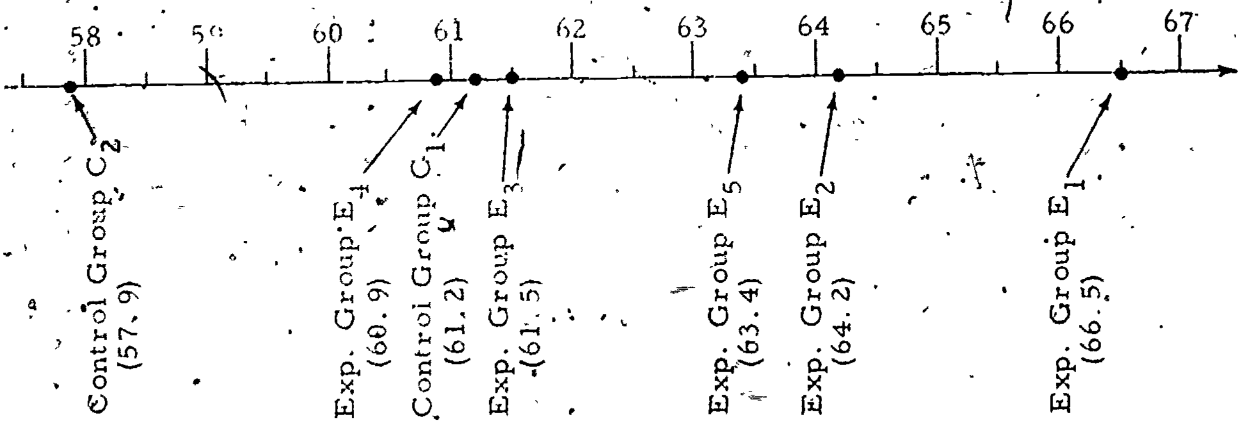

Figure 2. Adjusted Coop Algebra means for five experimental and two control groups (adjustments based on regression equation for combined sample).

Note that these means do not agree with those reported earlier in different group-by-group comparisons. This is because the earlier adjustments were each based on a regression equation applicable to the particular pair of groups being compared, while the present adjustments are based on the total sample. These adjusted means do not enter into significance tests, but they give a rough idea of what the relative standings of the several groups might have been if all groups had been comparable in terms of DAT-VR and DAT-NA means.

Second, we show in Table 9 a list of all the comparisons between various groups that were made. An inequality sign in this table indicates whether the adjusted Coop-Algebra mean for the group named on the left margin was significantly greater than (>), significantly less than (<), or not significantly different from (=) than that for the other group.

Table 9. Comparisons of adjusted group means on Coop Algebra

Statistical test used: Standard Covariance Analysis

Groups Compared	Adjusted Group Means	F-ratio	P	Conclusions
E_{total} C_{total}	62.50 60.45	31.19	$< .001$	$E_{total} > C_{total}$
E_1 $E_{(2...6)}$	67.33 62.76	34.79	$< .001$	$E_1 > E_{(2...6)}$
$E_{(2...6)}$ C_{total}	61.85 60.04	14.90	$< .001$	$E_{(2...6)} > C_{total}$
E_2 C_{total}	62.41 58.99	17.57	$< .001$	$E_2 > C_{total}$
E_3 C_{total}	60.48 59.56	2.53	$> .05$	$E_3 \doteq C_{total}$
E_4 C_{total}	59.07 58.68	.67	$> .05$	$E_4 \doteq C_{total}$
E_2 E_4	63.49 60.20	15.63	$< .001$	$E_2 > E_4$
E_3 E_5	63.21 66.75	11.86	$< .01$	$E_5 > E_3$

Table 9. (continued)

Statistical test used: Unrestricted Linear Hypothesis Model[3]

Compared	Differences Between Adjusted Means	Standard Error of Estimate	t-ratio	P	Conclusions: Computer Program due to Watson*	Conclusions: Johnson-Neyman Technique**
C_1 / C_2	1.11	2.93	.38	$> .05$	$C_1 \doteq C_2$	
E_5 / C_{total}	3.00	.538	5.57	$< .001$	$E_5 > C_{total}$	$E_5 > C_{total}$ ($P < .05$)
E_3 / C_1	6.92	2.33	2.97	$< .01$	$E_3 > C_1$	$E_3 \doteq C_1$ ($P > .05$)

*See page 13 **See page 16

[3]See page 13